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A COMMUNICATION SYSTEM AND METHOD FOR PERFORMING FAST SYMBOL ESTIMATION FOR MULTIPLE ACCESS DISPERSE CHANNELS

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Patent Application

A COMMUNICATION SYSTEM AND METHOD FOR PERFORMING FAST SYMBOL ESTIMATION FOR MULTIPLE ACCESS DISPERSE CHANNELS

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BACKGROUND OF THE INVENTION

The present invention relates to wireless and wireline communication systems. More specifically, the present invention relates to an improved method for estimating symbol sequences of multiple sources sharing the same communication media. The invention is particularly useful for code-division-multiple-access (CDMA) receivers where the channel length is only limited to two neighboring symbols.

In a scenario where m receivers are utilized, the received signal at each receiver is the sum of the output of channels driven by u different user signals plus the noise at the receiver. The channel between receiver i and user j can be denoted as h_{ij}. In general, $h_{ij} \neq h_{qn}$ if $i \neq q$ or $j \neq n$. Noise signals at different receivers are assumed to be independent of one another. If the user signals are actually CDMA signals and the largest duration of h_{ij}, $i = 1, \cdots, m$ and $j = 1, \cdots, u$ is short enough compared to the symbol duration of those CDMA signals, the orthogonality of the spreading codes can guarantee the separation of CDMA signals at each receiver. Intersymbol interference (ISI) and multiple access interference (MAI) are negligible. When the largest duration of the signature waveforms is comparable with, or even larger than the symbol duration, ISI and MAI can become severe. The performance of detection made on symbols individually will not be good due to ISI and MAI. Such a situation calls for joint detection by which channel effects are modeled adequately and symbol sequences are estimated jointly. In minimum-mean-square-error (MMSE) based joint detection, which will be referred to as joint detection in the following, an inverse filtering process is used to remove intersymbol interference and multiple access interference. To do this, a coefficient matrix H formed from the channel responses is inverted. More precisely, the pseudo-inverse of this coefficient matrix: $(H^*H)^{-1}H^*$ must be found. The main problem associated with joint detection is that the computation thus involved is intensive. The dimension of H is determined by the number of users, the length of spreading sequences, the length of symbol sequences, and the number of receivers involved (even with only one receiver, the effect of multiple receivers can be achieved by sampling a received continuous signal at a multiple

times of the symbol rate.). With a moderate number of users and a moderate number of symbols, the dimension of the coefficient matrix can be large. To speed up the inversion of the coefficient matrix, a Cholesky decomposition on H^*H can be used. However, the computation amount involved in the Cholesky decomposition can still be substantial, as the requirement on matrix operations such as matrix inverses increases linearly with the number of symbols. It is therefore desirable to have a method to speed up the joint detection computation.

Prior to this invention, a class of multiuser detectors have been developed. The most prominent ones among many others include R. Lupas and Verdu, "Linear multiuser detectors for synchronous CDMA channels", IEEE Trans. on Information Theory, 1(35):123-136, January 1989.; Z. Xie et al. "A family of sub-optimum detectors for coherent multiuser communications", IEEE Journal of Selected Areas in Communications, pages 683-690, May 1990; and Z. Zvonar and D. Brady, "Suboptimum multiuser detector for syncrhonous CDMA frequency-selective Rayleigh fading channels", IEEE Trans. on Communications, Vol. 43 No. 2/3/4, pp.154-157, February/March/April, 1995, Angel M. Bravo, "Limited linear cancellation of multiuser interference in DS/CDMA asynchronous systems", pp. 1435-1443, November, 1997. They either design various finite-impulse-response filters to process the received signals, which are in essence approximate solutions of the MMSE based joint detection solution, or use Cholesky decomposition to solve the equations involving the large coefficient matrix, e.g. Paul D. Alexander and Lars Rasmussen, "On the windowed Cholesky factorization of the time-varying asynchronous CDMA channel", IEEE Trans. on communications, vol. 46, no. 6, pp.735-737, June, 1998, which gives the exact MMSE joint detection result with high computational complexity, i.e. complexity of matrix operations such as matrix inversion increases linearly with the number of symbols.

As in many communication systems, the length of ISI and MAI is often limited to several symbols, besides being Toeptliz or block Toeplitz matrix, the coefficient matrix is also banded. Fast algorithms developed for generic Toeplitz or block Toeplitz matrices, e.g. Georg Heinig and Karla Rost, "Algebraic Methods for Toeplitz-like Matrices and Operators", Birkhauser, 1984, do not take advantage of the fact that the coefficient matrix is banded. The afore-mentioned Cholesky factorization based algorithm, while taking advantage of the banded nature of the coefficient matrix, does not take advantage of the fact that the coefficient matrix is also Toeplitz or block Toeplitz, which leads to considerable increase of computational demand as the number of matrix operations such as matrix inversion increases with the number of symbols to be estimated. Simpler versions of the method disclosed in this invention, have been used to solve numerically Poisson equations, e.g. Roland A. Sweet, "A cyclic reduction algorithm for solving block tridiagonal systems of arbitrary dimension", SIAM journal on Numerical Analysis, Vol. 14, pp. 706-720, September, 1977.

SUMMARY OF THE INVENTION

The present invention discloses a symbol detection system and method for multiple users in scenarios where the medium is shared by multiple users. The symbol detection system

and method involve formulating the received signals as the product of the inverse of a block triadiagonal matrix and a vector formed by the symbol sequences. The block tridiagonal matrix is derived from the channel responses. The present invention includes a fast algorithm to find the product. The computational complexity of the disclosed method is linear with the number of symbols and the requirement for matrix operations such as matrix inversion increases with the logarithm of the number of symbols.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 is a graphic illustration of a wireless communications setup for the disclosed technique.
FIG. 2 is a graphic illustration of a wireline communications setup for the disclosed technique.
FIG. 3 is an overview of the disclosed algorithm.
FIG. 4 is a flow chart of the forward reduction procedure.
FIG. 5 is a flow chart of the backward substitution procedure.

While the invention is susceptible to various modifications and alternative forms, specific embodiments thereof are shown by way of example in the drawings and will herein be described in detail. It should be understood, however, that the drawings and detailed description thereto are not intended to limit the invention to the particular form disclosed, but on the contrary, the intention is to cover all modifications, equivalents and alternatives falling within the spirit and scope of the present invention as defined by the appended claims.

DETAILED DESCRIPTION OF THE DRAWINGS

INTRODUCTION

The basic problem under consideration is that of estimation of symbol sequences with intersymbol interference and multiple access interference. To give a concrete setup of the joint detection problem under study, we consider a wireless CDMA system with multiple antennas (receivers) at the base station, and consider the joint detection problem of finding the symbol sequences from these users, i.e., the joint detection problem of the uplink signals received at the base station. As was made clear previously, by oversampling, the effect of multiple virtual receivers can be achieved with only one physical receiver, the applicability of the procedures which will be given is apparent for the joint detection problem of downlink signals also. In a code division multiple access system with J users, there are M ($M \geq 1$) antennas and hence M physical receivers at the base station, and the length of the spreading sequences is P. The received signal at each physical receiver is oversampled at M_t times the chip rate thereby producing a plurality of received signals each sampled at the chip rate. In other words, for each physical receiver, the received signal is oversampled to produce a number of chip rate signals. Hence at the base station, the signal received at antenna i can be represented as follows:

$$r_i(n) = \sum_k h_{ik}(n) * c_k(n), \quad i = 1, \cdots M,$$

where $h_{ik}(n)$ is the signature waveform of the channel between the k-th user and the i-th antenna at the base station. The signature waveform can be identified either by blind methods or with training sequences, e.g. Tong et al., "Blind Identification and Equalization Based on Second-Order Statistics: A Time-Domain Approach", IEEE Trans. on Information Theory, March, 1994.

FIG. 1 shows one example of an embodiment where the invention may be used. A base station (**102**) receives signals from wireless terminals (**104, 106, 108, 110**). For some wireless terminals(**106, 108**), there is more than one path between them and and the base station, as made clear by the reflective paths by reflectors (**112, 114**). Path length difference among multiple paths can destroy the orthogonality of signal waveforms of different transmitters which may have existed. Methods based matched filters and simple despreading are not able to offer satisfactory results under such circumstances. MMSE joint detection can offer much better results with heavier computation complexity. The disclosed method is able to reduce the computation amount requirement for implementing MMSE joint detection considerably.

In the embodiment of FIG. 1, the symbol estimator of the invention may be comprised in the base station (**102**) for estimating symbols in uplink transmissions from wireless terminals (**104,106,108,110**). The symbol estimator of the invention may also be comprised in one or more of the wireless terminals (**104,106,108,110**) for estimating symbols in downlink transmission from base station (**102**).

FIG. 2 shows another embodiment of the disclosed invention. A cable modem node (**200**) receives signals from multiple cable modems (**206, 208, 210**) which share the same cable medium (**212**) to communicate with the cable modem node (**200**). The channels from different cable modems to the cable modem node (**200**) will be different. In the embodiment of FIG. 2, the symbol estimator of the invention may be comprised in the cable modem node (**200**). Similar to the situation shown in FIG. 1, the disclosed technique can also be applied to speed up the joint detection method that combats the dispersive channel effects. It is not difficult to see that the invented technique can be applied in a reversed way, i.e., the cable modems (**206,208, 210**) may include the symbol estimator of the invention and may receive the signal from the cable modem node (**200**).

FIG. 3 shows one embodiment of the method used by this invention to estimate symbol sequences of multiple users. The base-band signals are collected and sampled from multiple receivers (**302**). In (**304**), At each receiver, the channel response between a transmitter and that receiver is used to filter the sampled received signal at that receiver. As there are J channel responses between all the transmitters and that receiver, there are a total of J filtered outputs produced from the sampled received signal at receiver. All the outputs are multiplexed to form a single data vector, according the transmitter indices, in an ascending order. Next all the multiplexed data vectors from all the receivers are added up to form one single vector Y_1 (**318**).

Channel responses $h_{mj}(n)$, $m = 1, \cdots, M$, $j = 1, \cdots, J$ (**312**) between transmitters

and receivers are identified either by training sequences or other methods which are not specified in this invention. Space-time correlation matrices $\mathbf{T}_i$, $i = d,\ d-1,\ \cdots,\ -(d-1),\ -d$, are constructed from those channel responses (**312**). Channel description matrices $\mathbf{A}_1$, $\mathbf{B}_1$, and $\mathbf{D}_i$ are block Toeplitz matrices, the blocks of which are defined by $\mathbf{T}_i$, $i = d,\ d-1,\ \cdots,\ -(d-1),\ -d$, which are constructed to form the description matrices (**316**). Forward reduction steps (**306**) are initialized with the second set of matrices and vectors, i.e. $\mathbf{A}_1,\ \mathbf{B}_1, \mathbf{D}_1$ and Y_1. In the course of forward reduction, using the bisection approach, some intermediate matrices which will be used in the backward substitution steps in (**308**) are stored. At the beginning of backward substitution steps (**308**), the intermediate solution X_s is found. Solved unknowns and intermediate matrices which have been stored in the forward reduction steps are used to find other unknowns. The output of the backward substitution steps (**308**) is demultiplexed (**310**) to produce the estimate of symbol sequences sent by transmitters.

FIG. 4 is a flow chart showing the forward reduction procedure.

FIG.5 is a flow chart showing the backward substitution procedure.

DATA FORMATION

In this disclosure, cases where signature waveforms of arbitrary duration are considered. In the following, we will write the received signals as the convolution of signature waveforms and oversampled symbol sequences in a structural way, which facilitates the derivation of the fast algorithm. If the signature waveform duration is limited to $d+1$ symbol duration, then we can pad zeros to $h_{i,k}(n)$ so that the length of $h_{i,k}(n)$ is precisely $(d+1)P$. Assume that $h_{mj}(n)$, $1 \le m \le M$, $1 \le j \le J$ are column vectors, which are identified with either training sequences or other means, which is not specified in this invention (**312**). We first define some notation. We divide $h_{mj}(n)$ into $(d+1)$ parts:

$$h_{mj} = \begin{bmatrix} h_1^{(mj)} \\ h_2^{(mj)} \\ \vdots \\ h_{d+1}^{(mj)} \end{bmatrix},$$

where $h_j^{(mj)}$, $j = 1,\ \cdots,\ d+1$, are $P \times 1$ vectors. We put together the signature waveform vectors:

$$\begin{bmatrix} \overbrace{h_{m1} \quad \cdots \quad h_{mJ}}^{J} \end{bmatrix} = \begin{bmatrix} \overbrace{\begin{matrix} h_1^{(m1)} & \cdots & h_1^{(mJ)} \\ h_2^{(m1)} & \cdots & h_2^{(mJ)} \\ \vdots & \vdots & \vdots \\ h_{d+1}^{(m1)} & \cdots & h_{d+1}^{(mJ)} \end{matrix}}^{J} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1^{(m)} \\ \mathbf{H}_2^{(m)} \\ \vdots \\ \mathbf{H}_{d+1}^{(m)} \end{bmatrix},$$

where

$$\mathbf{H}_k^{(m)} := \left[\overbrace{h_j^{(m1)} \quad \cdots \quad h_j^{(mJ)}}^{J} \right],$$

$\mathbf{H}_\mathbf{k}^{(\mathrm{m})}$ is a $P \times J$ matrix, $k = 1, \cdots, d+1$.

Put the received signal at receiver m into a vector fashion:

$$\underbrace{\begin{bmatrix} L+d \left\{ \begin{matrix} r_m(1) \\ r_m(2) \\ \vdots \\ r_m((L+d)P) \end{matrix} \right. \end{bmatrix}}_{R_m} = \underbrace{\begin{bmatrix} {\scriptstyle L+d} \left\{ \begin{matrix} \overbrace{\begin{matrix} \mathbf{H}_1^{(m)} & & & & \\ \ddots & \ddots & & & \\ \mathbf{H}_{d+1}^{(m)} & \ddots & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & \ddots \\ & & & \ddots & \ddots & \mathbf{H}_1^{(m)} \\ & & & & \ddots & \ddots \\ & & & & & \mathbf{H}_{d+1}^{(m)} \end{matrix}}^{L} \end{matrix} \right. \end{bmatrix}}_{\mathbf{H}_m} \underbrace{\begin{bmatrix} x_1(1) \\ x_2(1) \\ \vdots \\ x_J(1) \\ \vdots \\ x_1(L) \\ x_2(L) \\ \vdots \\ x_j(L) \end{bmatrix}}_{X}.$$

By stacking the received signals: R_m, $m = 1, \cdots, M$, one over another, we have

$$\underbrace{\begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_M \end{bmatrix}}_{R} = \underbrace{\begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_M \end{bmatrix}}_{\mathbf{H}} X + \underbrace{\begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_M \end{bmatrix}}_{W}. \tag{1}$$

The objective of joint detection is to find X. If

$$(L+1)PM \geq LJ, \tag{2}$$

then normally $\mathbf{H}$ is a full rank matrix.

The zero forcing solution is given by

$$\hat{X} = (\mathbf{H}^*\mathbf{H})^{-1}\mathbf{H}^*R = (\sum_{m=1}^{M} \mathbf{H}_m^*\mathbf{H}_m)^{-1} \sum_{m=1}^{M} \mathbf{H}_m^* R_m. \tag{3}$$

The MMSE (minimum mean square error) solution is given by

$$\hat{X} = (\mathbf{H}^*\mathbf{H} + \sigma^2\mathrm{I})^{-1}\mathbf{H}^*R = (\sum_{m=1}^{M} \mathbf{H}_m^*\mathbf{H}_m + \sigma^2\mathrm{I})^{-1} \sum_{m=1}^{M} \mathbf{H}_m^* R_m, \tag{4}$$

where σ^2 is the variance of the noise. We define a matrix:

$$\mathbf{T} := \mathbf{H}^*\mathbf{H} + v\mathbf{I} = \sum_{m=1}^{M} \mathbf{H}_m^*\mathbf{H}_m + v\mathbf{I},$$

and a vector

$$Y := \sum_{m=1}^{M} \mathbf{H}_m^* R_m,$$

where v can be either 0 or σ^2 depending on whether the zero-forcing solution or the MMSE solution is in need. $\mathbf{H}_m^* R_m$, $m = 1, \cdots, M$ are formed in (304). Y is formed in (318). In both zero-forcing and MMSE solutions, the computation of $\hat{X}$ can be separated in two steps:

1. Find Y.
2. Find $\mathbf{T}^{-1}Y$.

In either case, $\mathbf{T}$ is a block banded matrix, defined in the following way,

$$\mathbf{T} = \begin{bmatrix} \mathbf{T}_0 & \ddots & \mathbf{T}_{-d} & & & & & & \\ \ddots & \ddots & \ddots & \ddots & & & & & \\ \mathbf{T}_d & \ddots & \ddots & \ddots & \ddots & & & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & & & \\ & & \ddots & \ddots & \ddots & \ddots & \ddots & & \\ & & & \ddots & \ddots & \ddots & \ddots & \ddots & \mathbf{T}_{-d} \\ & & & & \ddots & \ddots & \ddots & \ddots & \ddots \\ & & & & & \mathbf{T}_d & \ddots & \ddots & \mathbf{T}_0 \end{bmatrix}_{LJ \times LJ}, \tag{5}$$

where

$$\mathbf{T}_i = \sum_{m=1}^{M} \sum_{k=0}^{d+i} (\mathbf{H}_{d+1-k}^{(m)})^* \mathbf{H}_{d+1-k+i}^{(m)} + \delta(i) v\mathbf{I}, \quad \mathbf{T}_{-i} = \mathbf{T}_i^*, \quad i = 0, \ -1 \ \cdots, \ -d.$$

And $\mathbf{T}_i, i = d, \ d-1, \cdots, \ -(d-1), \ -d$ are called space-time correlation matrices (314), $delta(\cdot)$ is the Kronecker function.

We can represent $\mathbf{T}$ as a block tridiagonal matrix, if d divides L:

1. the diagonal block $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{bmatrix} \mathbf{T}_0 & \ddots & \ddots & \mathbf{T}_{-(d-1)} \\ \ddots & \ddots & \ddots & \ddots \\ \ddots & \ddots & \ddots & \ddots \\ \mathbf{T}_{d-1} & \ddots & \ddots & \mathbf{T}_0 \end{bmatrix}_{dJ \times dJ},$$

2. the supra-diagonal block $\mathbf{B}$ is given by

$$\mathbf{B} = \begin{bmatrix} \mathbf{T}_{-d} & & & \\ \ddots & \ddots & & \\ \ddots & \ddots & \ddots & \\ \mathbf{T}_{-1} & \ddots & \ddots & \mathbf{T}_{-d} \end{bmatrix}_{dJ \times dJ}.$$

We can write

$$\mathbf{T}\hat{X} = Y \tag{6}$$

as

$$L\left\{\begin{array}{c} \overbrace{\hspace{5cm}}^{L} \\ \begin{bmatrix} \mathbf{A} & \mathbf{B} & & & & \\ \mathbf{B}^* & \mathbf{A} & \mathbf{B} & & & \\ & \ddots & \ddots & \ddots & & \\ & & \ddots & \ddots & \ddots & \\ & & & \mathbf{B}^* & \mathbf{A} & \mathbf{B} \\ & & & & \mathbf{B}^* & \mathbf{A} \end{bmatrix} \end{array}\right. \hat{X} = Y. \tag{7}$$

FAST JOINT DETECTOR

DERIVATION

The system of equations as given in (7) can be treated as the special case of a more general formulation defined by a triplet [$\mathbf{A}$, $\mathbf{B}$, $\mathbf{D}$]:

$$L_i\left\{\begin{array}{c} \overbrace{\hspace{5cm}}^{L_i} \\ \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i & & & & \\ \mathbf{B}^*{}_i & \mathbf{A}_i & \mathbf{B}_i & & & \\ & \ddots & \ddots & \ddots & & \\ & & \ddots & \ddots & \ddots & \\ & & & \mathbf{B}^*{}_i & \mathbf{A}_i & \mathbf{B}_i \\ & & & & \mathbf{B}^*{}_i & \mathbf{D}_i \end{bmatrix} \end{array}\right. X_i = Y_i, \tag{8}$$

where $\mathbf{A}_i$, $\mathbf{B}_i$, and $\mathbf{D}_i$ are $J \times J$ matrices, which are called channel description matrices (**316**),

$$X_i = \begin{bmatrix} X_{i,1} \\ \vdots \\ X_{i,L_i}, \end{bmatrix}, \tag{9}$$

$$Y_i = \begin{bmatrix} Y_{i,1} \\ \vdots \\ Y_{i,L_i}, \end{bmatrix}, \tag{10}$$

with $X_{i,j}$ and $Y_{i,j}$ being $J \times 1$ vectors.

We linearly combine equations for the following cases:

1. for $2k = 2$ (i.e. $k = 1$),

$$\begin{array}{lcc} 2k-1: & -\mathbf{B^*}_i\mathbf{A}_i^{-1} & \times \\ 2k: & \mathbf{I} & \times \\ 2k+1: & -\mathbf{B}_i\mathbf{A}_i^{-1} & \times \end{array} \left(\begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i & & \\ \mathbf{B^*}_i & \mathbf{A}_i & \mathbf{B}_i & \\ & \mathbf{B^*}_i & \mathbf{A}_i & \mathbf{B}_i \end{bmatrix} \begin{bmatrix} X_{i,2k-1} \\ X_{i,2k} \\ X_{i,2k+1} \\ X_{i,2k+2} \end{bmatrix} = \begin{bmatrix} Y_{i,2k-1} \\ Y_{i,2k} \\ Y_{i,2k+1} \end{bmatrix} \right), \tag{11}$$

and we have

$$\begin{bmatrix} \mathbf{0} & -\mathbf{B^*}_i\mathbf{A}_i^{-1}\mathbf{B}_i + \mathbf{A}_i - \mathbf{B}_i\mathbf{A}^{-1}\mathbf{B^*}_i & \mathbf{0} & -\mathbf{B}_i\mathbf{A}_i^{-1}\mathbf{B}_i \end{bmatrix} \begin{bmatrix} X_{i,2k-1} \\ X_{i,2k} \\ X_{i,2k+1} \\ X_{i,2k+2}, \end{bmatrix} \tag{12}$$

$$= -\mathbf{B^*}_i\mathbf{A}_i^{-1}Y_{i,2k-1} + Y_{i,2k} - \mathbf{B}_i\mathbf{A}_i^{-1}Y_{i,2k+1},$$

or

$$\begin{aligned} &\begin{bmatrix} -\mathbf{B^*}_i\mathbf{A}_i^{-1}\mathbf{B}_i + \mathbf{A}_i - \mathbf{B}_i\mathbf{A}^{-1}\mathbf{B^*}_i & -\mathbf{B}_i\mathbf{A}_i^{-1}\mathbf{B}_i \end{bmatrix} \begin{bmatrix} X_{i,2k} \\ X_{i,2k+2} \end{bmatrix} \\ &= -\mathbf{B^*}_i\mathbf{A}_i^{-1}Y_{i,2k-1} + Y_{i,2k} - \mathbf{B}_i\mathbf{A}_i^{-1}Y_{i,2k+1}. \end{aligned} \tag{13}$$

2. For every even index $2k (4 \leq 2k \leq L_i - 3)$, we can linearly combine the $2k-1^{st}$, $2k^{th}$, and $2k+1^{st}$ equations in system of equations (8):

$$\begin{array}{lcc} 2k-1: & -\mathbf{B^*}_i\mathbf{A}_i^{-1} & \times \\ 2k: & \mathbf{I} & \times \\ 2k+1: & -\mathbf{B}_i\mathbf{A}_i^{-1} & \times \end{array} \left(\begin{bmatrix} \mathbf{B^*}_i & \mathbf{A}_i & \mathbf{B}_i & & \\ & \mathbf{B^*}_i & \mathbf{A}_i & \mathbf{B}_i & \\ & & \mathbf{B^*}_i & \mathbf{A}_i & \mathbf{B}_i \end{bmatrix} \begin{bmatrix} X_{i,2k-2} \\ X_{i,2k-1} \\ X_{i,2k} \\ X_{i,2k+1} \\ X_{i,2k+2} \end{bmatrix} = \begin{bmatrix} Y_{i,2k-1} \\ Y_{i,2k} \\ Y_{i,2k+1} \end{bmatrix} \right), \tag{14}$$

and we have

$$\begin{aligned} &\begin{bmatrix} -\mathbf{B^*}_i\mathbf{A}_i^{-1}\mathbf{B^*}_i & \mathbf{0} & -\mathbf{B^*}_i\mathbf{A}_i^{-1}\mathbf{B}_i + \mathbf{A}_i - \mathbf{B}_i\mathbf{A}^{-1}\mathbf{B^*}_i & \mathbf{0} & -\mathbf{B}_i\mathbf{A}_i^{-1}\mathbf{B}_i \end{bmatrix} \begin{bmatrix} X_{i,2k-2} \\ X_{i,2k-1} \\ X_{i,2k} \\ X_{i,2k+1} \\ X_{i,2k+2} \end{bmatrix} \\ &= -\mathbf{B^*}_i\mathbf{A}_i^{-1}Y_{i,2k-1} + Y_{i,2k} - \mathbf{B}_i\mathbf{A}_i^{-1}Y_{i,2k+1}, \end{aligned} \tag{15}$$

or

$$
\begin{aligned}
&\begin{bmatrix} -\mathbf{B}^*{}_i\mathbf{A}_i^{-1}\mathbf{B}^*{}_i & -\mathbf{B}^*{}_i\mathbf{A}_i^{-1}\mathbf{B}_i + \mathbf{A}_i - \mathbf{B}_i\mathbf{A}^{-1}\mathbf{B}^*{}_i & -\mathbf{B}_i\mathbf{A}_i^{-1}\mathbf{B}_i \end{bmatrix} \begin{bmatrix} X_{i,2k-2} \\ X_{i,2k} \\ X_{i,2k+2} \end{bmatrix} \\
= \; & -\mathbf{B}^*{}_i\mathbf{A}_i^{-1}Y_{i,2k-1} + Y_{i,2k} - \mathbf{B}_i\mathbf{A}_i^{-1}Y_{i,2k+1}.
\end{aligned} \tag{16}
$$

3. Depending on whether L_i is odd or even, we need to study two different cases.

   (a) If L_i is odd, the last even index $2k$ between 1 and L_i is $L_i - 1$. Let $L_i = 2j+1$, so $k = j$. We combine the last three equations

$$
\begin{matrix} 2j-1: & -\mathbf{B}^*{}_i\mathbf{A}_i^{-1} & \times \\ 2j: & \mathbf{I} & \times \\ 2j+1: & -\mathbf{B}_i\mathbf{D}_i^{-1} & \times \end{matrix} \left(\begin{bmatrix} \mathbf{B}^*{}_i & \mathbf{A}_i & \mathbf{B}_i & \\ & \mathbf{B}^*{}_i & \mathbf{A}_i & \mathbf{B}_i \\ & & \mathbf{B}^*{}_i & \mathbf{D}_i \end{bmatrix} \begin{bmatrix} X_{i,2j-2} \\ X_{i,2j-1} \\ X_{i,2j} \\ X_{i,2j+1} \end{bmatrix} = \begin{bmatrix} Y_{i,2j-1} \\ Y_{i,2j} \\ Y_{i,2j+1} \end{bmatrix} \right), \tag{17}
$$

   we have

$$
\begin{aligned}
&\begin{bmatrix} -\mathbf{B}^*{}_i\mathbf{A}_i^{-1}\mathbf{B}^*{}_i & 0 & -\mathbf{B}^*{}_i\mathbf{A}_i^{-1}\mathbf{B}_i + \mathbf{A}_i - \mathbf{B}_i\mathbf{D}_i^{-1}\mathbf{B}^*{}_i & 0 \end{bmatrix} \begin{bmatrix} X_{i,2j-2} \\ X_{i,2j-1} \\ X_{i,2j} \\ X_{i,2j+1} \end{bmatrix} \\
= \; & -\mathbf{B}^*{}_i\mathbf{A}_i^{-1}Y_{i,2j-1} + Y_{i,2j} - \mathbf{B}_i\mathbf{D}_i^{-1}Y_{i,2j+1},
\end{aligned} \tag{18}
$$

   or

$$
\begin{aligned}
&\begin{bmatrix} -\mathbf{B}^*{}_i\mathbf{A}_i^{-1}\mathbf{B}^*{}_i & -\mathbf{B}^*{}_i\mathbf{A}_i^{-1}\mathbf{B}_i + \mathbf{A}_i - \mathbf{B}_i\mathbf{D}^{-1}\mathbf{B}^*{}_i \end{bmatrix} \begin{bmatrix} X_{i,2j-2} \\ X_{i,2j} \end{bmatrix} \\
= \; & -\mathbf{B}^*{}_i\mathbf{A}_i^{-1}Y_{i,2j-1} + Y_{i,2j} - \mathbf{B}^*{}_i\mathbf{D}_i^{-1}Y_{i,2j+1}.
\end{aligned} \tag{19}
$$

   (b) If L_i is even, the last even index $2k$ between 1 and L_i is L_i. Let $L_i = 2j$. We combine the last two equations:

$$
\begin{matrix} 2j-1: & -\mathbf{B}^*{}_i\mathbf{A}_i^{-1} & \times \\ 2j: & \mathbf{I} & \times \end{matrix} \left(\begin{bmatrix} \mathbf{B}^*{}_i & \mathbf{A}_i & \mathbf{B}_i \\ & \mathbf{B}^*{}_i & \mathbf{D}_i \end{bmatrix} \begin{bmatrix} X_{i,2j-2} \\ X_{i,2j-1} \\ X_{i,2j} \end{bmatrix} = \begin{bmatrix} Y_{i,2j-1} \\ Y_{i,2j} \end{bmatrix} \right), \tag{20}
$$

   we have

$$
\begin{bmatrix} -\mathbf{B}^*{}_i\mathbf{A}_i^{-1}\mathbf{B}^*{}_i & 0 & -\mathbf{B}^*{}_i\mathbf{A}_i^{-1}\mathbf{B}_i + \mathbf{D}_i \end{bmatrix} \begin{bmatrix} X_{i,2j-2} \\ X_{i,2j-1} \\ X_{i,2j} \end{bmatrix} = -\mathbf{B}^*{}_i\mathbf{A}_i^{-1}Y_{i,2j-1} + Y_{i,2j}, \tag{21}
$$

or

$$\left[\begin{array}{cc} -\mathbf{B^*}_i\mathbf{A}_i^{-1}\mathbf{B^*}_i & -\mathbf{B^*}_i\mathbf{A}_i^{-1}\mathbf{B}_i + \mathbf{D}_i \end{array} \right] \left[\begin{array}{c} X_{i,2j-2} \\ X_{i,2j} \end{array} \right] = -\mathbf{B^*}_i\mathbf{A}_i^{-1}Y_{i,2j-1} + Y_{i,2j}. \tag{22}$$

To facilitate a structural description of the newly-generated equations, we can define the following:

$$\begin{aligned} \mathbf{A}_{i+1} &:= -\mathbf{B^*}_i\mathbf{A}_i^{-1}\mathbf{B}_i + \mathbf{A}_i - \mathbf{B}_i\mathbf{A}_i^{-1}\mathbf{B^*}_i, \\ \mathbf{B}_{i+1} &:= -\mathbf{B}_i\mathbf{A}_i^{-1}\mathbf{B}_i. \end{aligned} \tag{23}$$

If L_i is odd, we define

$$\mathbf{D}_{i+1} := \mathbf{A}_i - \mathbf{B}_i\mathbf{D}_i^{-1}\mathbf{B^*}_i - \mathbf{B^*}_i\mathbf{A}_i^{-1}\mathbf{B}_i, \tag{24}$$

and

$$L_{i+1} := \frac{1}{2}(L_i - 1). \tag{25}$$

If L_i is even, we define

$$\mathbf{D}_{i+1} := \mathbf{D}_i - \mathbf{B^*}_i\mathbf{A}_i^{-1}\mathbf{B}_i, \tag{26}$$

and

$$L_{i+1} := \frac{1}{2}L_i. \tag{27}$$

We also define

$$X_{i+1,k} := X_{i,2k}, \quad 1 \leq 2k \leq L_i. \tag{28}$$

$\mathbf{A}_{i+1}$, $\mathbf{B}_{i+1}$, and $\mathbf{D}_{i+1}$ are the new channel description matrices.

Substitute the above newly-defined variables into equations (16) and (13) for $1 \leq 2k \leq L_i - 2$, and equation (19) if L_i is odd, or equation (22) if L_i is even. We obtain the following system of equations:

$$L_{i+1}\left\{ \left[\overbrace{\begin{array}{cccccc} \mathbf{A}_{i+1} & \mathbf{B}_{i+1} & & & & \\ \mathbf{B^*}_{i+1} & \mathbf{A}_{i+1} & \mathbf{B}_{i+1} & & & \\ & \ddots & \ddots & \ddots & & \\ & & \ddots & \ddots & \ddots & \\ & & & \mathbf{B^*}_{i+1} & \mathbf{A}_{i+1} & \mathbf{B}_{i+1} \\ & & & & \mathbf{B^*}_{i+1} & \mathbf{D}_{i+1} \end{array}}^{L_{i+1}} \right] \right. X_{i+1} = Y_{i+1}. \tag{29}$$

We call the system of equations given in (29) as the $i+1^{st}$ system of equations. Hence a system of equations of reduced dimension involving only $X_{i,j}$ of even indices as unknowns is obtained in the cancellation process, and Y_{i+1} is obtained. It can be seen that the coefficient matrix defined by the triplet $[\mathbf{A}_{i+1},\ \mathbf{B}_{i+1},\ \mathbf{D}_{i+1}]$ is still a block-tridiagonal matrix, even though of smaller dimension than the coefficient matrix defined by by the triplet $[\mathbf{A}_i,\ \mathbf{B}_i,\ \mathbf{D}_i]$. Hence the same procedure can be again applied to this new system of equations. With the execution of the procedure each time, the size of the system of equations is reduced by at

least a half. If we start from the 1^{st} system of equations as given in (10), by executing this procedure not more than $log_2(L_1)$ times, we can generate the 2^{nd}, 3^{rd}, $\cdots$ systems of equations. Eventually, at some s, the $s-1^{th}$ system of equations would be given either as

$$\begin{bmatrix} \mathbf{A}_{s-1} & \mathbf{B}_{s-1} & \\ \mathbf{B}^*{}_{s-1} & \mathbf{A}_{s-1} & \mathbf{B}_{s-1} \\ & \mathbf{B}^*{}_{s-1} & \mathbf{D}_{s-1} \end{bmatrix} X_{s-1} = Y_{s-1}, \tag{30}$$

or

$$\begin{bmatrix} \mathbf{A}_{s-1} & \mathbf{B}_{s-1} \\ \mathbf{B}^*{}_{s-1} & \mathbf{D}_{s-1} \end{bmatrix} X_{s-1} = Y_{s-1}. \tag{31}$$

(30) and (31) can be reduced to the s^{th} system of equations involving one $u \times 1$ subvector: $X_{s,1}$ as unknown, respectively through (17) and (20):

$$\mathbf{D}_s X_s = Y_s, \tag{32}$$

we have

$$X_s = \mathbf{D}_s^{-1} Y_s. \tag{33}$$

The just-found $X_{s,1}$(or $X_{s-1,2}$ as in the $s-1^{st}$ system of equations) can be substituted into the $s-1^{st}$ system of equations. In general, for the i^{th} system of equations, from solution of the $i+1^{st}$ system of equations, $X_{i,j}$ of even indices are already known, and $X_{i,j}$ of odd indices are still unknown. We have three cases to consider.

1. Consider the first equation in the i^{th} system of equations:

$$\begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i \end{bmatrix} \begin{bmatrix} X_{i,1} \\ X_{i,2} \leftarrow X_{i+1,1} \end{bmatrix} = Y_{i,1}, \tag{34}$$

   where $X_{i,2}(X_{i+1,1})$ has been found from the solution of the $i+1^{st}$ system of equations. We have

$$X_{i,1} = \mathbf{A}_i^{-1}(Y_{i,1} - \mathbf{B}_{i,1} X_{i+1,1}). \tag{35}$$

2. Consider the $2k+1^{st}$ equation in the i^{th} system of equations, $k = 1,\ 2$, etc.:

$$2k+1: \qquad \begin{bmatrix} \mathbf{B}^*{}_i & \mathbf{A}_i & \mathbf{B}_i \end{bmatrix} \begin{bmatrix} X_{i,2k} \leftarrow X_{i+1,k} \\ X_{i,2k+1} \\ X_{i,2k+2} \leftarrow X_{i+1,k+1} \end{bmatrix} = Y_{i,2k+1}, \tag{36}$$

   where the values of $X_{i,2k}(X_{i+1,k})$ and $X_{i,2k+2}(X_{i+1,k+1})$ have been found from the solution of the $i+1^{st}$ system of equations. We have

$$\mathbf{A}_i X_{i,2k+1} = Y_{i,2k+1} - \mathbf{B}^*{}_i X_{i+1,k} - \mathbf{B}_i X_{i+1,k+1}, \tag{37}$$

   or

$$X_{i,2k+1} = \mathbf{A}_i^{-1}(Y_{i,2k+1} - \mathbf{B}^*{}_i X_{i+1,k} - \mathbf{B}_i X_{i+1,k+1}). \tag{38}$$

3. If L_i is odd($L_i = 2k + 1$), then X_{i,L_i} is found from the last block equation:

$$\left[\begin{array}{cc} \mathbf{B^*}_i & \mathbf{D}_i \end{array} \right] \left[\begin{array}{c} X_{i,L_i-1} \\ X_{i,L_i} \end{array} \right] = Y_{i,L_i} \tag{39}$$

with

$$X_{i,L_i} = \mathbf{D}_i^{-1}(Y_{i,L_i} - \mathbf{B^*}_i X_{i,L_i-1}). \tag{40}$$

Consequently for the i^{th} system of equations, we just solve the block equations corresponding to odd indices. Hence all the unknowns in the i^{th} system of equations are found. The above procedure can be recursively applied to the $i - 1^{st}$, $i - 2^{nd}$, $\cdots$ systems of equations until the unknowns in the system of equations as given in (7) are all found.

Apparently, the main computation is on the right-side of the equations. The update of the triplets involves light computation.

In summary, the solution of the block tridiagonal system of equations as given in (10) is executed with two procedures. In the first procedure, the size of the system of equations is reduced, which is called forward reduction (**306**), a flow chart of which is shown in FIG. 4. In the second procedure, solved unknowns are substituted into larger systems of equations and more unknowns are solved, which is called backward substitution (**308**), a flow chart of which is shown in FIG. 5.

COMPUTATIONAL PROCEDURE

The whole procedure is naturally separated into two parts: the forward reduction procedure and the backward substitution procedure.

The forward reduction procedure:

1. $\mathbf{A}_1 := \mathbf{A}$, $\mathbf{B}_1 := \mathbf{B}$, $\mathbf{D}_1 := \mathbf{A}$, $Y_1 := Y$, $L_1 := L$, $i := 1$.

2. Find the inverse of $\mathbf{A}_i$: $\mathbf{A}_i^{-1}$.

3. $\mathbf{Q}_i := \mathbf{B}_i\mathbf{A}_i^{-1}$ and $\mathbf{E}_i := \mathbf{B^*}_i\mathbf{A}_i^{-1}$.

4. $\mathbf{A}_{i+1} := \mathbf{A}_i - \mathbf{E}_i\mathbf{B}_i - (\mathbf{E}_i\mathbf{B}_i)^*$.

5. $\mathbf{B}_{i+1} := -\mathbf{Q}_i\mathbf{B}_i$.

6. • If L_i is odd,
   (a) $\mathbf{P}_i := \mathbf{B}_i\mathbf{D}_i^{-1}$,
   (b) $\mathbf{D}_{i+1} := \mathbf{A}_i - \mathbf{P}_i\mathbf{B^*}_i - \mathbf{E}_i\mathbf{B}_i$,
   (c) $L_{i+1} := \frac{L_i - 1}{2}$,
   (d) $Y_{i+1,k} := -\mathbf{E}_i Y_{i,2k-1} + Y_{i,2k} - \mathbf{Q}_i Y_{i,2k+1}$, $1 \le k < L_{i+1}$,
   (e) $Y_{i+1,L_{i+1}} := -\mathbf{E}_i Y_{i,L_i-2} + Y_{i,L_i-1} - \mathbf{P}_i Y_{i,L_i}$.

   • If L_i is even,

(a) $\mathbf{D}_{i+1} := \mathbf{D}_i - \mathbf{E}_i\mathbf{B}_i$,
(b) $L_{i+1} := \frac{L_i}{2}$,
(c) $Y_{i+1,k} := -\mathbf{E}_i Y_{i,2k-1} + Y_{i,2k} - \mathbf{Q}_i Y_{i,2k+1}$, $1 \leq k < L_{i+1}$,
(d) $Y_{i+1,L_{i+1}} := -\mathbf{E}_i Y_{i,L_i-1} + Y_{i,L_i}$.

7. Store $\mathbf{A}_{i+1}^{-1}$, $\mathbf{D}_{i+1}^{-1}$, $\mathbf{Q}_i$, $\mathbf{E}_i$, $\mathbf{P}_i$ (if it is calculated), and Y_i.

8. $i := i + 1$.

9. Repeat 2 to 8 until $L_i = 1$.

10. $s := i$.

A flow chart of the above-stated procedure is given in FIG. 4.
The backward substitution procedure:

1. Solve $\mathbf{D}_s X_s = Y_s$.

2. $i := s - 1$.

3. $X_{i,2k} := X_{i+1,k}$, $1 \leq k \leq L_{i+1}$.

4. $j := 1$.

5. $X_{i,j} := \begin{cases} \mathbf{A}_i^{-1} Y_{i,1} - \mathbf{E}_i^* X_{i,j+1}, & \text{if } j = 1. \\ -\mathbf{Q}_i^* X_{i,j-1} + \mathbf{A}_i^{-1} Y_{i,j} - \mathbf{E}_i^* X_{i,j+1}, & \text{if } 1 < j < L_i. \\ -\mathbf{P}_i^* X_{i,j-1} + \mathbf{D}_i^{-1} Y_{i,j}, & \text{if } j = L_i. \end{cases}$

6. $j := j + 2$, if $j < L_i$ go to 5.

7. $i := i - 1$, if $i > 0$ go to 3.

8. X_1 is then the estimate of X.

A flow chart of the above-stated procedure is given in FIG. 5.

Although the system and method of the present invention has been described in connection with the preferred embodiment, it is not intended to be limited to the specific form set forth herein, but on the contrary, it is intended to cover such alternatives, modifications, and equivalents, as can be reasonably included within the spirit and scope of the invention as defined by the appended claims.